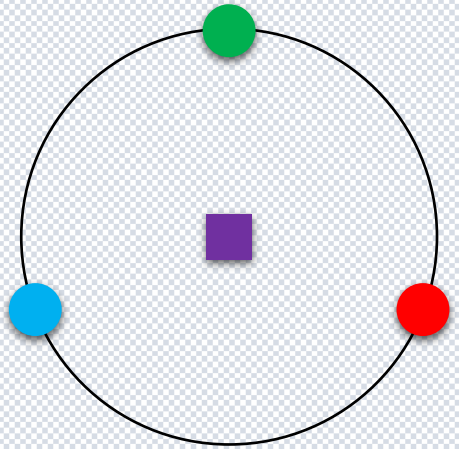
Northeastern University

Stealthy Optimal Range-Sensor Placement for Target Localization

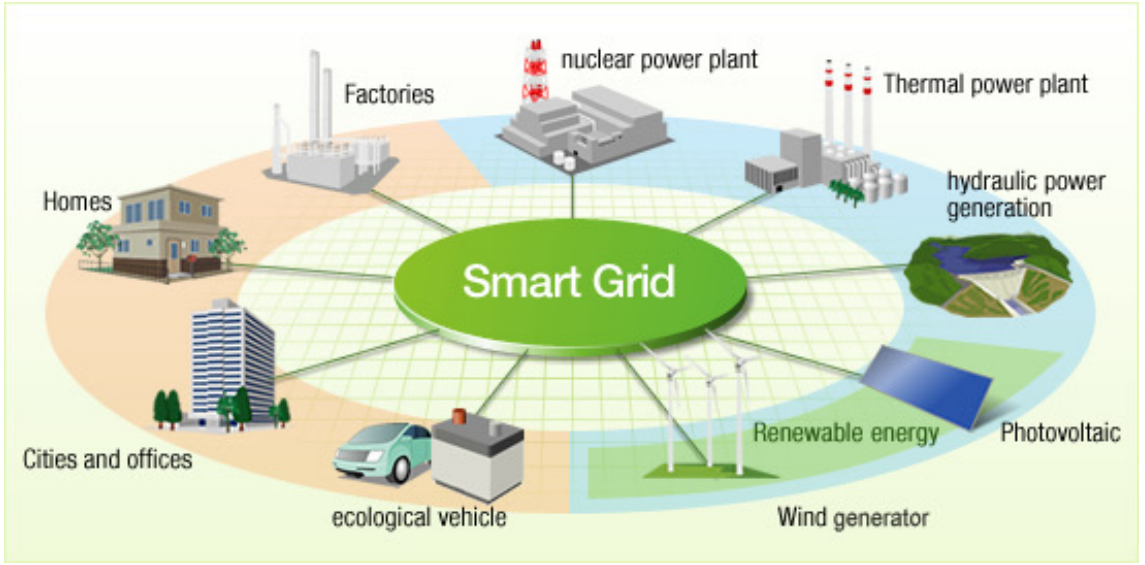


Mohammad Hussein Yoosefian Nooshabadi, Rifat Sipahi and Laurent Lessard

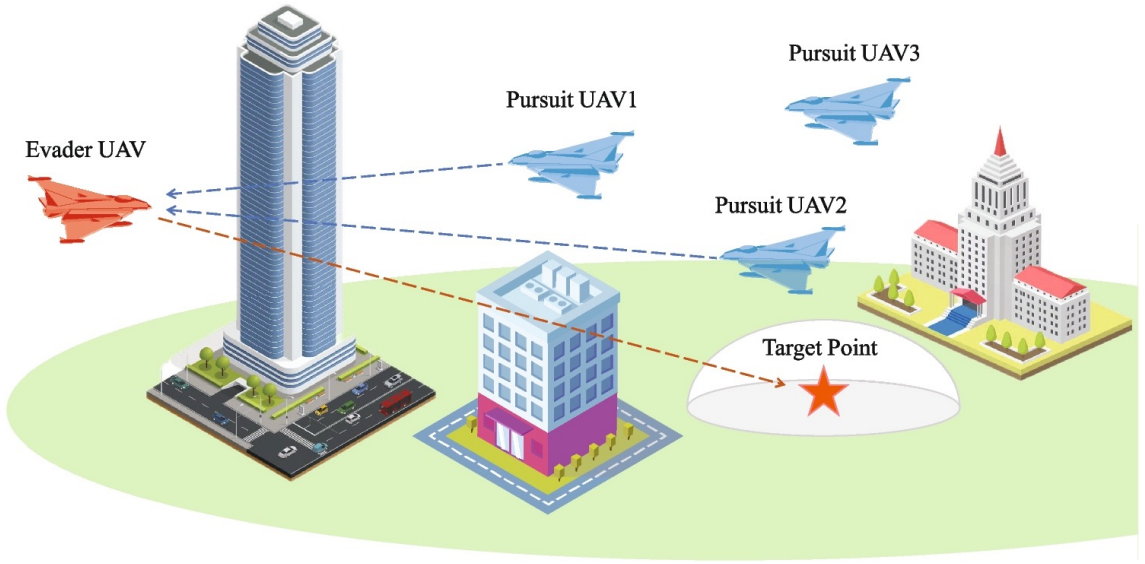
2025 American Control Conference

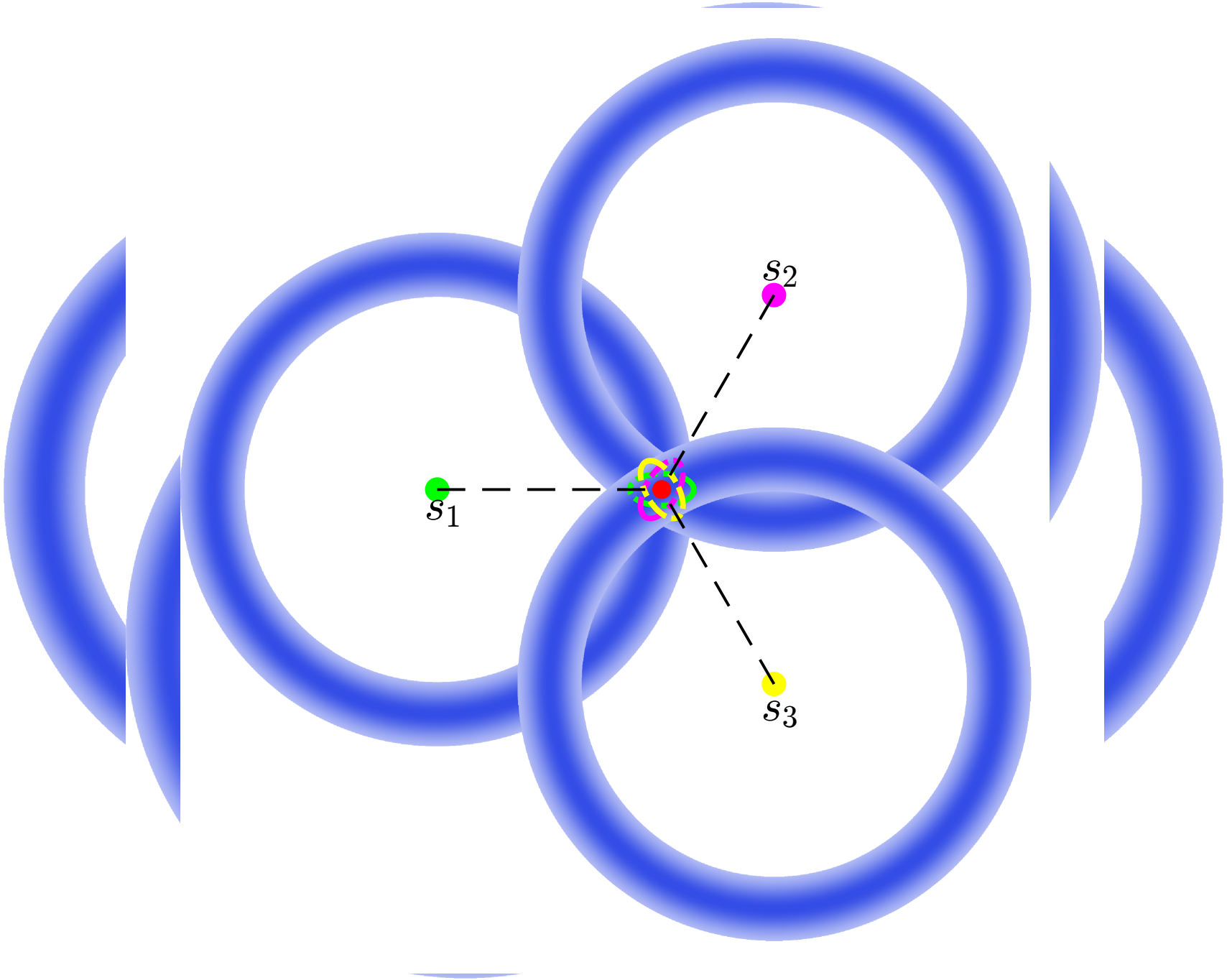
July 2025

- Target localization (Optimal sensor placement)
  - Search and rescue
  - Structural health monitoring
- Stealthy task accomplishment (Privacy and security)
  - Pursuit and evasion
  - Smart grids
- Goal: Bridge between the two literatures
  - Privacy-preserving surveillance
  - Distributed sensing and security



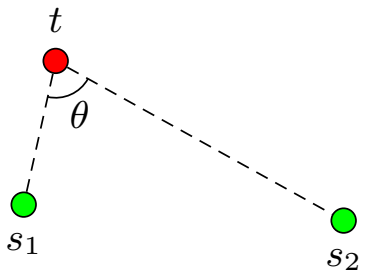





s_2
s_1
s_3

Encoding Information [Sadeghi, 2020]

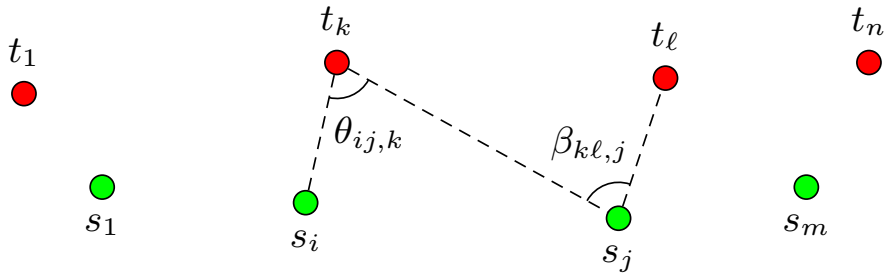
$$z_i = \|x_t - x_{s_i}\| + v, \quad v \sim \mathcal{N}(0, \sigma^2) \longrightarrow \mathcal{I} = \frac{1}{\sigma^4} \sin^2 \theta$$



Problem Formulation

$$\mathcal{I}_k = \frac{1}{\sigma^4} \sum_{1 \le i < j \le m} \sin^2 \theta_{ij,k}$$

$$\mathcal{J}_i = \frac{1}{\sigma^4} \sum_{1 \le k < \ell \le n} \sin^2 \beta_{k\ell,i}$$



$$\underset{\theta,\,\beta}{\text{maximize}} \quad \min_{1 \le k \le n} \mathcal{I}_k$$

$$\text{subject to:} \quad \max_{1 \le i \le m} \mathcal{J}_i \le \gamma^2$$

$$(\theta, \beta) \in \mathcal{F}$$

- $\mathcal{I}_k$, $\mathcal{J}_i$: amount of information
- $\mathcal{F}$: set of feasible angles
- γ^2: information leakage level


t_2
t_4
t_3
t_1
$\gamma = 0.5$

Two Sensors and Two Targets

The problem

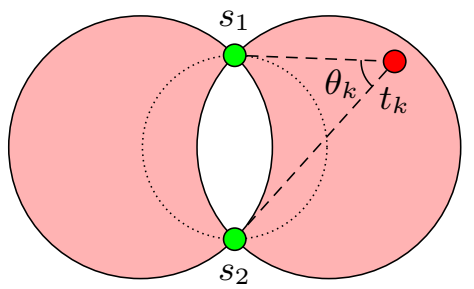
$$\underset{\theta_1,\theta_2,\beta_1,\beta_2}{\text{maximize}} \quad \min\left(\sin^2\theta_1,\ \sin^2\theta_2\right) \qquad \text{(a)}$$

$$\text{subject to:} \quad \max\left(\sin^2\beta_1,\ \sin^2\beta_2\right) \le \gamma^2 \qquad \text{(b)}$$

$$(\theta_1,\theta_2,\beta_1,\beta_2) \in \mathcal{F} \qquad \text{(c)}$$

(a) Objective function

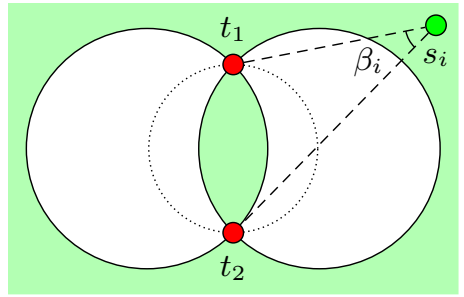
$$\min\left(\sin^2\theta_1,\ \sin^2\theta_2\right) \ge \eta^2 \longrightarrow \theta_k \in [\arcsin\eta,\ \pi - \arcsin\eta] \quad \text{for } k = 1,2$$



$\eta = 0.7$ shown

(b) Stealthiness constraint

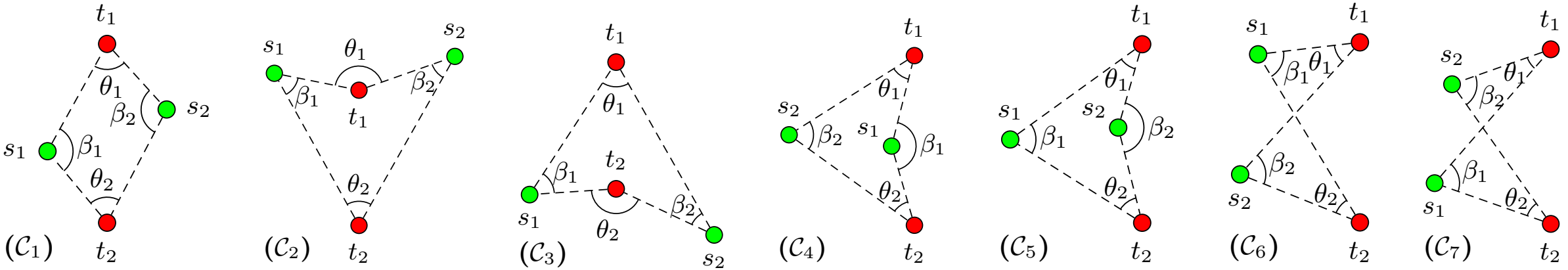
$$\max\left(\sin^2\beta_1,\ \sin^2\beta_2\right) \le \gamma^2 \longrightarrow \beta_i \in [0,\arcsin\gamma] \cup [\pi - \arcsin\gamma,\pi] \quad \text{for } i = 1,2$$



$\gamma = 0.7$ shown

Two Sensors and Two Targets

(c) Set of feasible angles



$$
\begin{aligned}
\mathcal{C}_1 &: \quad \theta_1 + \theta_2 + \beta_1 + \beta_2 = 2\pi \\
\mathcal{C}_2 &: \quad -\theta_1 + \theta_2 + \beta_1 + \beta_2 = 0, \quad \theta_2 \le \theta_1 \\
\mathcal{C}_3 &: \quad \theta_1 - \theta_2 + \beta_1 + \beta_2 = 0, \quad \theta_1 \le \theta_2 \\
\mathcal{C}_4 &: \quad \theta_1 + \theta_2 - \beta_1 + \beta_2 = 0, \quad \beta_2 \le \beta_1 \\
\mathcal{C}_5 &: \quad \theta_1 + \theta_2 + \beta_1 - \beta_2 = 0, \quad \beta_1 \le \beta_2 \\
\mathcal{C}_6 &: \quad \theta_1 - \theta_2 + \beta_1 - \beta_2 = 0, \quad \theta_1 + \beta_1 \le \pi \\
\mathcal{C}_7 &: \quad \theta_1 - \theta_2 - \beta_1 + \beta_2 = 0, \quad \theta_1 + \beta_2 \le \pi
\end{aligned}
\quad \longrightarrow \quad
\mathcal{F} = \left\{ (\theta_1, \theta_2, \beta_1, \beta_2) \in [0, \pi]^4 \;\middle|\; \bigcup_{i=1}^{7} \mathcal{C}_i \right\}
$$

Two Sensors and Two Targets

- Theorem 1 (sensors can be freely placed): A configuration is globally optimal iff:

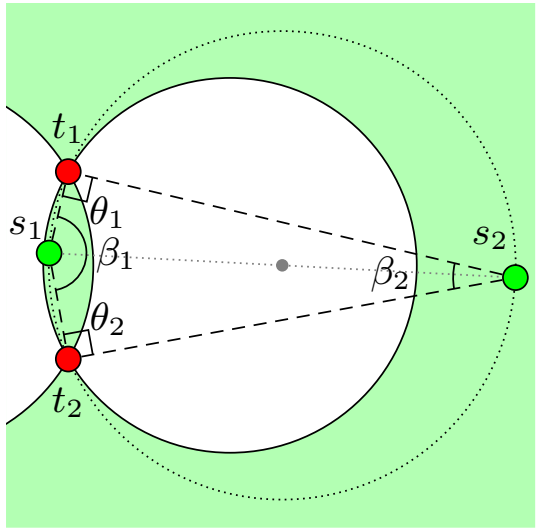
  (i) s_1, s_2, t_1, t_2 are *cyclic* (they lie on a common circle),

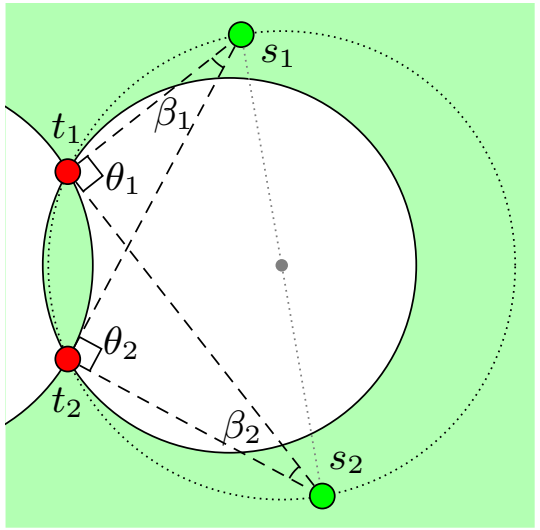
  (ii) s_1 and s_2 are diametrically opposed on this circle, and

  (iii) The common circle has diameter at least d/γ; where d is the distance between t_1 and t_2.

$$\begin{aligned} \underset{\theta_1,\theta_2,\beta_1,\beta_2}{\text{maximize}} \quad & \min\left(\sin^2\theta_1,\ \sin^2\theta_2\right) \\ \text{subject to:} \quad & \max\left(\sin^2\beta_1,\ \sin^2\beta_2\right) \le \gamma^2 \\ & (\theta_1,\theta_2,\beta_1,\beta_2) \in \mathcal{F} \end{aligned}$$

$\theta_1 = \theta_2 = \frac{\pi}{2}$ and $\sin\beta_1 = \sin\beta_2$ and the optimal objective value is 1.





Two Sensors and Two Targets

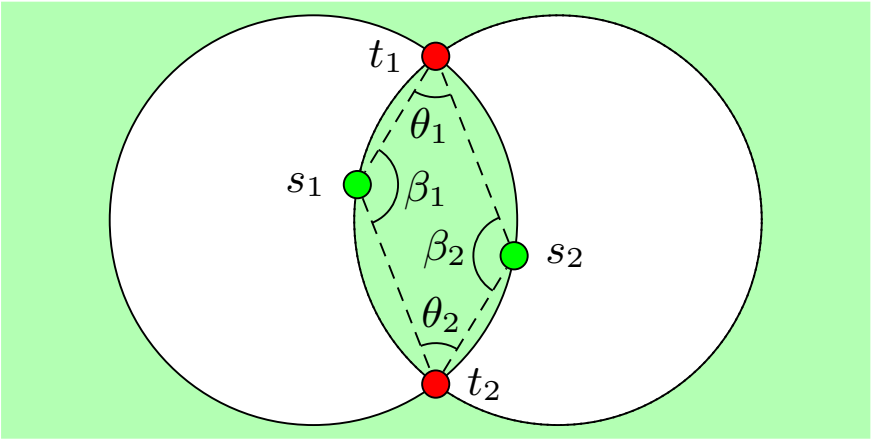
- Theorem 2 (sensors are restricted to be inside): A configuration is globally optimal iff:

  (i) s_1, t_1, s_2 and t_2 are vertices of a parallelogram, and

  (ii) s_1 and s_2 are on different circles through t_1 and t_2 with diameter d/γ; where d is the distance between t_1, t_2.

$\theta_1 = \theta_2 = \arcsin\gamma$ and $\beta_1 = \beta_2 = \pi - \arcsin\gamma$ and the optimal objective value is γ^2.

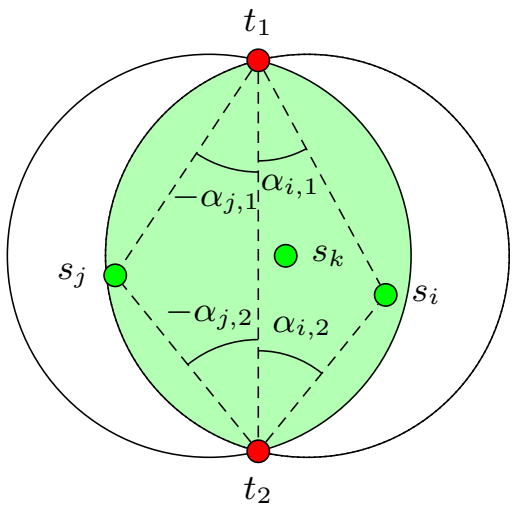
$$\begin{aligned} \underset{\theta_1,\theta_2,\beta_1,\beta_2}{\text{maximize}} \quad & \min\left(\sin^2\theta_1,\ \sin^2\theta_2\right) \\ \text{subject to:} \quad & \max\left(\sin^2\beta_1,\ \sin^2\beta_2\right) \le \gamma^2 \\ & (\theta_1,\theta_2,\beta_1,\beta_2) \in \mathcal{F} \\ & \beta_i \in [\pi - \arcsin\gamma, \pi] \text{ for } i = 1,2 \end{aligned}$$



Arbitrarily Many Sensors and Two Targets

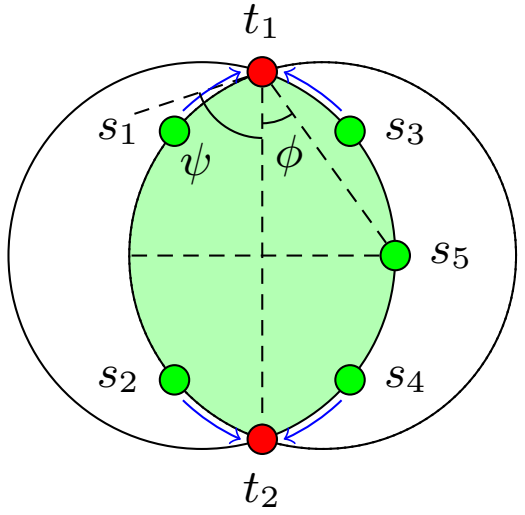
The problem

$$\begin{aligned}
&\underset{\alpha,\eta^2}{\text{maximize}} && \eta^2 && \\
&\text{subject to:} && \sum_{1\le i<j\le m} \sin^2(\alpha_{i,k}-\alpha_{j,k}) \ge \eta^2 && k\in\{1,2\} \\
& && |\alpha_{i,1}+\alpha_{i,2}| \le \arcsin\gamma && i\in\{1,\dots,m\} \\
& && \alpha_{i,1}\alpha_{i,2} \ge 0 && i\in\{1,\dots,m\}
\end{aligned}$$



- Analytical bounds
  - Degenerate configuration (lower bound)

$$\eta^2_{\text{opt}} \ge \begin{cases}
\frac{1}{4}m^2\gamma^2(2-\gamma^2) & \text{if } m\equiv 0 \pmod 4 \\
\frac{1}{4}(m-1)\left(2-2(1-\gamma^2)^{3/2}+(m-1)\gamma^2(2-\gamma^2)\right) & \text{if } m\equiv 1 \pmod 4 \\
\frac{1}{4}m^2\gamma^2(2-\gamma^2)-\gamma^2+\gamma^4 & \text{if } m\equiv 2 \pmod 4 \\
\frac{1}{4}(m-1)\left(2-2(1-\gamma^2)^{3/2}+(m-1)\gamma^2(2-\gamma^2)\right) & \\
\qquad -\gamma^2+\gamma^4+\gamma^2\sqrt{1-\gamma^2} & \text{if } m\equiv 3 \pmod 4
\end{cases}$$

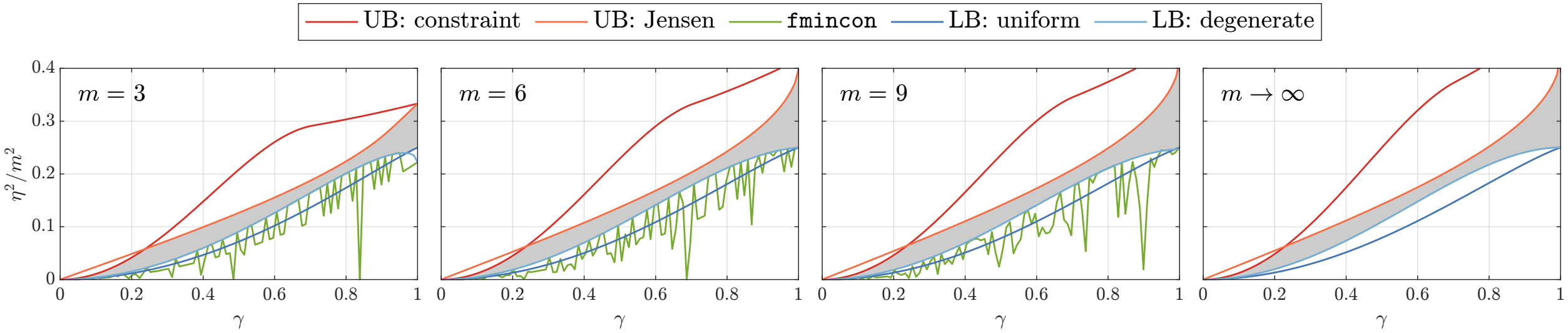


$$\psi = 2\phi = \arcsin\gamma$$

Arbitrarily Many Sensors and Two Targets

- Other analytical bounds
  - Uniform lower bound
  - Constraint upper bound
  - Jensen's upper bound

- Combining the bounds



Conclusion and Future Works

Conclusion

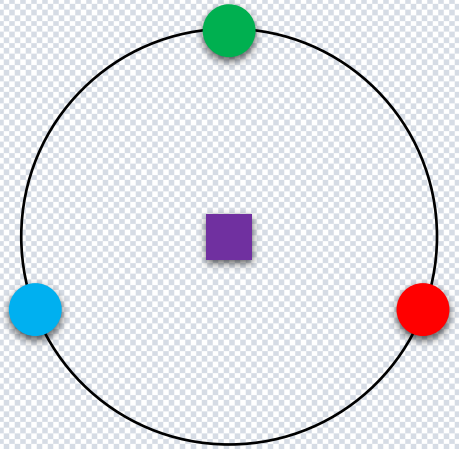
- Stealthy optimal range-sensor placement for target localization
- Analytical solution for two sensors and two targets
- Upper and lower bounds for arbitrarily many sensors and two targets

Future Work

- Other sensor models; e.g., bearing only
- Moving targets and sensors; i.e., target tracking
- Case with more targets



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Thank you for your attention!