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# Formation flight control and path tracking of a multi-quadrotor system in the presence of measurement noise and disturbances

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***Abstract*— This paper focuses on formation flight control of a multi-quadrotor system. Path tracking of this system is another subject which is in the spot light. When a fault occurs for leader of the group in the leader-follower structure, mission will be failed. In order to avoid this fault, in this paper virtual structure is used instead. The group uses two control loops for flight controlling. Inner loop which is used to control attitudes of agents, combines feedback linearization controller with PD controller. Outer loop which is designed for position control, uses a combination of consensus protocol with PID controller. There is measurement noise in the system, thereby exact values of the states are not available. In order to compensate these negative factor, Kalman filter is used, thus states values are estimated. Finally, effectiveness of the presented protocol is examined through stability analysis and numerical simulation.**

***Keywords*— Quadrotor, Path tracking, formation flight control, feedback linearization, PID controller***

## I. INTRODUCTION

Nowadays, formation movement of autonomous vehicles; such as aircrafts, has become one of the most attractive research fields in control engineering. Thanks to advances in embedded systems technology, network infrastructures, and control techniques, large groups of automated agents can cooperate to perform the assigned tasks. Such systems are called multi-agent systems. In most of these assignments, it is desirable to put the group members in distinctive geometric shapes known as formations. Some most important applications of aeronautical multi-agent systems in operational conditions are formation flight for surveillance and identification tasks, heavy load transportation, aerial refueling, and satellites coordinated arrangement. Moreover, quadrotors have caught a lot of individuals' attention from scholars to companies, due to the great advantages and abilities of these types of UAVs. Thus, coordinated flight of quadrotors for performing assigned tasks; such as surveillance and identification and etc., is one of the most attractive areas of research. Experimental path planning for several quadrotors, has been performed in GRASP lab in University of Pennsylvania [1-2] and SCORE lab in University of Macau [3]. In [1], path planning in presence of obstacles has also been investigated. In [2], formation of different aerial and ground robots has been discussed . A cluster of unmanned aerial vehicles has been used for cooperative surveillance on a wide ground in [3]. In Max Planck institute a Tele-Operating multi-UAV system has been evaluated [4-5]. Heudiasyc lab has presented a scenario to search for individuals by a group of quadrotors in a large area and in a Leader-follower structure [6]. All of these mentioned works are done practically, without considering the specific theory in the field of controlling multi-agent systems.

In [7] flight formation control of multiple mini rotorcraft systems has been achieved by using nonlinear controller based on separated saturations. In this work, simplified equations of motion of quadrotors and leader-follower structure have been used. In 2011, consensus protocols have been used in a multi-quadrotor system for environmental coverage based on location optimization techniques [8]. Simplification of equations and planner movement of quadrotors have been assumed in that work. In [9], formation control of several quadrotors has been analyzed by linearizing of equations of quadrotors and based on a consensus algorithm and leader-follower structure. Works on formation control of several quadrotors can be seen in [10-13], where equations of quadrotors have been linearized or simplified. These researches have used consensus algorithms based on dynamic surface control and $H_\infty$ controller and leader-follower structure. Formation control and path planning based on inverse dynamic and back stepping controller has been used in [14]. The Euler angles have been assumed to be small in this research. Several quadrotors with a specific formation have been analyzed in [15]. In this work, a nonlinear cooperative controller based on dynamic surface control and leader-follower structure has been used. Path tracking and obstacle avoidance have been evaluated in [16] based on fuzzy logic. Formation control of several quadrotors using sliding mode controller has been discussed in [17] with the small Euler angles assumption. In [18], two loops have been used for controlling a group of quadrotors. First loop guarantees the stability of each quadrotor based on Lyapunov theory and the other loop leads the group to the target point. Consensus problem for several quadrotors by using feedback linearization has been discussed in [19]. Linearized model of the quadrotors and formation control of those have been used in [20]. That research have used decentralized model and predictive control based on leader-follower structure.

Most researches in formation control of the quadrotors have used the leader-follower structure. The most significant drawback of this type of structures is mission failure when the leader of the group encounters a problem. In order to overcome to this problem, artificial structure is used for leader. Using of linearized model or simplified model are another limitations of above mentioned researches. In this paper complete model of the

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quadrotor is used without any simplification. Furthermore, new consensus algorithm based on PID and feedback linearization controllers is used for formation control of several quadrotors.

## II. QUADROTOR DYNAMICS

dynamic equations of linear motions and rotational motions of quadrotors are achieved using Newton second law and Euler-Lagrange equations respectively as same as those used in [21]. However, the simplification assumptions used in [21] are not used in this paper. $X^T = [x \quad y \quad z]$ indicates the position of a quadrotor in inertial coordination. $\Omega^T = [p \quad q \quad r]$ denotes the absolute angular velocity of the quadrotor in the body coordination. $\omega^T = [\phi \quad \theta \quad \psi]$ exhibits Euler angles of the quadrotor. The four angular speeds $\omega_i$ of the motors are real inputs to the system, but because of the particular type of modeling used in this research, it is advantageous to consider system inputs like (1).

$$\begin{aligned} U_1 &= b(\omega_1^2 + \omega_3^2 + \omega_2^2 + \omega_4^2) \\ U_2 &= bL(\omega_4^2 - \omega_2^2) \\ U_3 &= bL(\omega_3^2 - \omega_1^2) \\ U_4 &= d(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \end{aligned} \tag{1}$$

where $b$ and $d$ are lift force and drag torque coefficients respectively, and $L$ is length of the quadrotor arms. Considering (1) and equations stated in [21], dynamic equations of a quadrotor are like (2), (3) and (4).

$$\begin{aligned} \ddot{x} &= (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi)\frac{U_1}{m} \\ \ddot{y} &= (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)\frac{U_1}{m} \\ \ddot{z} &= (\cos\phi \cos\theta)\frac{U_1}{m} - g \end{aligned} \tag{2}$$

$$\begin{aligned} \dot{p} &= \frac{U_2}{I_x} - \frac{I_R}{I_x}(gu)q + qr\left(\frac{I_y - I_z}{I_x}\right) \\ \dot{q} &= \frac{U_3}{I_y} + \frac{I_R}{I_y}(gu)p + pr\left(\frac{I_z - I_x}{I_y}\right) \\ \dot{r} &= \frac{U_4}{I_z} + pq\left(\frac{I_x - I_y}{I_z}\right) \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{\phi} &= p + (\sin\phi \tan\theta)q + (\cos\phi \tan\theta)r \\ \dot{\theta} &= (\cos\phi)q - (\sin\phi)r \\ \dot{\psi} &= \left(\frac{\sin\phi}{\cos\theta}\right)q + \left(\frac{\cos\phi}{\cos\theta}\right)r \end{aligned} \tag{4}$$

where $g$ is the gravity acceleration and $m$ is mass of the quadrotor and $gu = (-\omega_1 + \omega_2 - \omega_3 + \omega_4)$ in (3).

According to (2), (3) and (4), three linear movements of the quadrotor are affected by the quadrotor's rotation while linear movements do not affect rotational movements. Therefore, two control loops are used to control the quadrotor. Outer loop, control the position of the quadrotor in directions of $x$, $y$ and $z$, and determine the desired $\theta$ , $\phi$ and $\psi$. Inner loop, control the attitude and altitude of the quadrotor.

## III. FORMATION CONTROL

### A. Outer loop control

It is assumed that there is $N$ quadrotors, and it is intended to control the formation flight of these quadrotors. Position controller outputs of the quadrotors in directions of $x$ and $y$ are considered as (5) and (6).

$$\begin{aligned} u_{xi} = k_{px1i}\Bigg( & r_{1i}\sum_{j\epsilon N_i} a_{ij}(x_i - x_j) \\ & + r_{2i}\sum_{j\epsilon N_i} a_{ij}(\dot{x}_i - \dot{x}_j)\Bigg) + k_{px2i}(x_0 + \Delta_{xi} - x_i) \\ & + k_{dxi}(\dot{x}_0 - \dot{x}_i) + \sum_{j\epsilon N_i} a_{ij}(\ddot{x}_i - \ddot{x}_j) + \ddot{x}_0 \end{aligned} \tag{5}$$

$$\begin{aligned} u_{yi} = k_{py1i}\Bigg( & r_{1i}\sum_{j\epsilon N_i} a_{ij}(y_i - y_j) \\ & + r_{2i}\sum_{j\epsilon N_i} a_{ij}(\dot{y}_i - \dot{y}_j)\Bigg) + k_{py2i}\left(y_0 + \Delta_{yi} - y_i\right) \\ & + k_{dyi}\left(\dot{y}_0 - \dot{y}_i\right) + \sum_{j\epsilon N_i} a_{ij}(\ddot{y}_i - \ddot{y}_j) + \ddot{y}_0 \end{aligned} \tag{6}$$

where $k_{px1i}, r_{1i}, r_{2i}, k_{px2i}, k_{py1i}, k_{py2i}, k_{dxi}$ and $k_{dyi}$ are the controller parameters which will be defined later. To prove the correctness of the proposed controllers, it is assumed that the position vector of $i_{th}$ quadrotor is $X_i = \{x_i \quad y_i\}^T$. Position vector of the group is considered as (7).

$$X = \{X_1{}^T \quad X_2{}^T \quad \dots \quad X_N{}^T\}^T \tag{7}$$

Let the position error vector of the $i_{th}$ quadrotor to be $E_i = \{e_{xi} \quad e_{yi}\}^T$ ,then the position error vector of the group will be considered as (8).

$$E = \{e_1{}^T \quad e_2{}^T \quad \dots \quad e_N{}^T\}^T \tag{8}$$

Each element of the position error vector of the $i_{th}$ quadrotor is defined as (9).

$$e_{xi} \cong \sum_{j\epsilon N_i} a_{ij}(x_i - x_j) + g_i(x_0 + \Delta_{xi} - x_i) \tag{9}$$

where $g_i$ is the relation of each quadrotor with the virtual leader. If the $i_{th}$ quadrotor receives information from the virtual leader, $g_i = 1$, otherwise $g_i = 0$. Derivatives of the errors stated in (9) are shown in (10).

$$\begin{aligned} \dot{e}_{xi} &= \sum_{j\epsilon N_i} a_{ij}(\dot{x}_i - \dot{x}_j) + g_i(\dot{x}_0 - \dot{x}_i) \\ \ddot{e}_{xi} &= \sum_{j\epsilon N_i} a_{ij}(\ddot{x}_i - \ddot{x}_j) + g_i(\ddot{x}_0 - \ddot{x}_i) \end{aligned} \tag{10}$$

The position error vector of the group is derived from the position vector of the group and showed in (11).

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$$E = -(L \otimes I_n)X + (G \otimes I_n)(X_0 - X) \Rightarrow$$
$$E = \big((L+G) \otimes I_n\big)(X_0 - X) \quad (11)$$

where $L$ is Laplacian matrix of the communication graph, $\otimes$ is Kronocker product and $G$ is a matrix with $g_i$ as its elements on its diagonal. In this paper, all of the quadrotors gets information from virtual leader, thus $G = I_N$. $n$ is the number of states of an agent and $X_0$ is position vector of the virtual leader which is defined as (12).

$$X_0 = \{x_0 + \Delta_{1x} \quad y_0 + \Delta_{1y} \quad \dots \quad x_0 + \Delta_{Nx} \quad y_0 + \Delta_{Ny}\}^T \quad (12)$$

By defining the control parameters in the form of $k_{px1\mathrm{i}} r_{1\mathrm{i}} = k_{px2\mathrm{i}} = k_{py1\mathrm{i}} r_{1\mathrm{i}} = k_{py2\mathrm{i}} \cong k_{p\mathrm{i}}$ and $k_{dx\mathrm{i}} = k_{dy\mathrm{i}} \cong k_{di}$, control parameters matrices can be considered as (13).

$$K_P = \mathrm{diag}\{k_{pi}\}$$
$$K_D = \mathrm{diag}\{k_{di}\} \quad (13)$$

By replacing (11) and (13) into (5) and (6), and a little simplification, the group position control vector is defined as (14).

$$U_X = (K_P \otimes I_n)E + (K_D \otimes I_n)\dot{E} + \big((L+G) \otimes I_n\big)\ddot{X}_0 + (L \otimes I_n)\ddot{X} \quad (14)$$

Where

$$U_X = \{u_{x1} \quad u_{y1} \quad u_{x2} \quad u_{y2} \quad \dots \quad \dots \quad u_{xN} \quad u_{yN}\}^T \quad (15)$$

It is obvious that we can define $U_X \cong \ddot{X}$. With replacement in (14), equation (16) is obtained.

$$\big((L+G) \otimes I_n\big)\big(\ddot{X}_0 - \ddot{X}\big) = (K_P \otimes I_n)E + (K_D \otimes I_n)\dot{E} \quad (16)$$

Double derivation of (11), will lead to the equation $\ddot{E} = \big((L+G) \otimes I_n\big)\big(\ddot{X}_0 - \ddot{X}\big)$. And by replacing it in (16) it can be written:

$$\ddot{E} + (K_D \otimes I_n)\dot{E} + (K_P \otimes I_n)E = 0 \quad (17)$$

If the control parameters are chosen as mentioned, all eigenvalues of the matrices $(K_P \otimes I_n)$ and $(K_D \otimes I_n)$ will be positive. The relation (18) is always established.

$$\lim_{t\to\infty} E = 0 \rightarrow \lim_{t\to\infty} e_1 \& e_2 \& \dots \& e_N = 0 \quad (18)$$

Therefore, $\{e_{xi} \quad e_{yi}\}^T$ in $i_{th}$ quadrotor goes to zero. Each one of the errors consists of two terms and because each term of these errors uses absolute values, both terms go to zero. This means that the $i_{th}$ quadrotor goes to the defined artificial leader and consensus is always maintained by other group agents too. By solving equation (17), equation (19) will be obtained.

$$E(t) = C_1 e^{M_1 t} + C_2 e^{M_2 t} \quad (19)$$

where $C_1$ and $C_2$ are constant parameters. $M_1$ and $M_2$ are achieved from (20).

$$M_1 = \frac{-(K_D \otimes I_n) - \sqrt{(K_D \otimes I_n)^2 - 4(K_P \otimes I_n)}}{2}$$
$$M_2 = \frac{-(K_D \otimes I_n) + \sqrt{(K_D \otimes I_n)^2 - 4(K_P \otimes I_n)}}{2} \quad (20)$$

The lowest rate of error convergence to zero is the $C_2 e^{M_2 t}$ of (19). Thus, in order to converge at a desired speed, the control parameters must be chosen in a way that the (21) is satisfied.

$$\{\min(\lambda_i(K_D \otimes I_n))\}^2 - 4\{\max(\lambda_i(K_P \otimes I_n))\} > 0 \quad (21)$$

where $\lambda_i$ are the eigenvalues of the matrices. The convergence rate of the error to zero is obtained from the equation (22).

$$\nu \le exp\left\{\frac{-\min(\lambda_i(K_D \otimes I_n)) + J}{2}\, t\right\} \quad (22)$$

where $\nu$ is the convergence rate of the error to zero and $J = \sqrt{\{\min(\lambda_i(K_D \otimes I_n))\}^2 - 4\{\max(\lambda_i(K_P \otimes I_n))\}}$. So far it has been proved that by choosing the proper control parameters, the group position error can be set to zero. But it requires the inner loop to be well designed. It should be noted that in order to remove the small deviation of the group from the desired path, one can add an integral term to controllers (5) and (6).

*B. inner loop controller*

Quadrotor is an under-actuated system and its instantaneous rotational angles are related to the quadrotor position. So, the desired rotational angles are obtained from the position controller of the quadrotor(i.e., Outer loop). Current position of the $i_{th}$ quadrotor and the virtual leader position are shown in figure 1. The distance between the current position of the $i_{th}$ quadrotor and the virtual leader position is defined as (23).

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$$
$$\psi_{di} = \arctan\left({}^{y_i - y_0}/_{x_i - x_0}\right) \quad (23)$$
$$\beta_i = \psi_{di} - \psi_i$$

where $d_i$ is distance and is defined as $d_i = \sqrt{(e_{xi})^2 + (e_{yi})^2}$. $\psi_{di}$ is the desired heading angle of the $i_{th}$ quadrotor. And $\beta_i$ is the error between the current heading and desired heading angle. Previously, it was demonstrated that $d_i$ goes to zero exponentially by using controllers (5) and (6). The force that must be applied to the $i_{th}$ quadrotor to reach the position of the virtual leader is obtained from (24).

$$\begin{bmatrix} f_{xi} \\ f_{yi} \\ f_{zi} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\varphi_i \sin\theta_i & \cos\varphi_i \cos\theta_i \\ 0 & \cos\varphi_i & -\sin\varphi_i \\ -\sin\theta_i & \sin\varphi_i \cos\theta_i & \cos\varphi_i \cos\theta_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ F_i \end{bmatrix} \quad (24)$$

where $\{f_{xi} \quad f_{yi} \quad f_{zi}\}^T$ is the applied force to the $i_{th}$ quadrotor, if there is $F_i$. The force $F_i$ is obtained from (25).

$$F_i = b \sum_{j=1}^{4} \omega_{ij}^2 \quad (25)$$

where $\omega_{ij}$ is $j_{th}$ motor speed of the $i_{th}$ quadrotor. By solving the (24) we have.

$$F_i \cos\varphi_i \sin\theta_i = f_{xi} = A_i \cos\beta_i$$
$$-F \sin\varphi_i = f_{yi} = A_i \sin\beta_i \quad (26)$$

where $A_i$ is desired force aligned with the line connecting the current position of the $i_{th}$ quadrotor to the image of the artificial leader's position on the $xy$ page. From equation (26), the desired

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rotational angles of the $i_{th}$ quadrotor to achieve the desired position, is obtained as follows:

$$\varphi_{di} = \text{Arcsin}\left(-\frac{A_i}{F_i}\sin\beta_i\right)$$
$$\theta_{di} = \text{Arcsin}\left(\frac{A_i\cos\beta_i}{F_i\cos\varphi_{di}}\right) \quad (27)$$

where $\frac{A_i}{F_i}$ is assumed to be the output of the position controllers of the $i_{th}$ quadrotor. In order to design inner controller, we assume that $\theta_i$ and $\varphi_i$ in (2) and (3) are small. Moreover, the terms which contain $I_R$ are neglected. By considering $\eta_i = \{\eta_{1i} \quad \eta_{2i} \quad \eta_{3i} \quad \eta_{4i}\}^T = \{z_i \quad \varphi_i \quad \theta_i \quad \psi_i\}^T$ as the states of the $i_{th}$ quadrotor, (2) and (3) can be written as:

$$\ddot{\eta}_i = f(\dot{\eta}_i) + s(\eta_i)U_i \quad (28)$$

Again, by considering as $H = \{\eta_1 \quad \eta_2 \quad \dots \quad \eta_N\}^T$, (28) turns to (29).

$$\ddot{H} = F(\dot{H}) + S(H)U \quad (29)$$

where $F(\dot{H})$ is a $4N \times 1$ vector obtained from $F(\dot{H}) = \{f(\dot{\eta}_1) \quad f(\dot{\eta}_2) \quad \dots \quad f(\dot{\eta}_N)\}^T$ and $S(H)$ is a $4N \times 4N$ matrix obtained from matrices of $s(\eta_i)$. A nonlinear controller is used as inner controller because of nonlinear terms in (29). Therefore, a feedback linearization controller is used.

$$U = \{S(H)\}^{-1} \times \left(F(\dot{H}) + V\right) \quad (30)$$

In order to avoid singularity in (30), $det\{S(H)\}$ must be nonzero. For $s(\eta_i)$ we have.

$$s(\eta_i) = \begin{bmatrix} \cos\eta_{2i}\cos\eta_{3i}/m_i & 0 & 0 & 0 \\ 0 & 1/I_{xi} & 0 & 0 \\ 0 & 0 & 1/I_{yi} & 0 \\ 0 & 0 & 0 & 1/I_{zi} \end{bmatrix} \quad (31)$$

It is assumed that $\theta_i$ and $\varphi_i$ are small. Therefore determinant of $s(\eta_i)$ is always nonzero. Consequently, determinant of $S(H)$ is always nonzero and presented controller (30) is always proper. By replacing (30) in (29) we have:

$$\ddot{H} = V \quad (32)$$

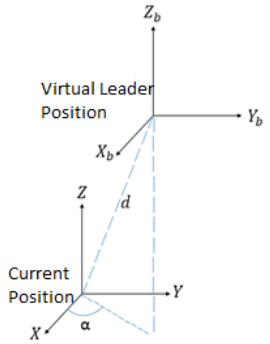



Fig. 1. Virtual leader and current position of one quadrotor

Desired states of each agent are $\eta_{di} = \{z_{di} \quad \varphi_{di} \quad \theta_{di} \quad \psi_{di}\}^T$. So group's desired states vector is $H_d$. By defining group states error as $\nabla = H_d - H$, a controller like (33) is proposed.

$$V = K_{p\eta}\nabla + K_{d\eta}\dot{\nabla} + \ddot{H}_d \quad (33)$$

Considering this proposed controller, (32) turns to (34).

$$\ddot{\nabla} + K_{d\eta}\dot{\nabla} + K_{p\eta}\nabla = 0 \quad (34)$$

By choosing $K_{d\eta}$ and $K_{p\eta}$ properly so that all eigenvalues of them are positive and satisfy (35) and (36), group states error goes to zero exponentially. Convergence rate of inner controller is larger than that of outer controller with these choices.

$$\{\min(\lambda_i(K_{d\eta}))\}^2 - 4\{\max(\lambda_i(K_{p\eta}))\} > 0 \quad (35)$$
$$\frac{-(\min(\lambda_i(K_{d\eta}))) + O}{2} < M_1 \quad (36)$$

where $O = \sqrt{\{\min(\lambda_i(K_{d\eta}))\}^2 - 4\{\max(\lambda_i(K_{p\eta}))\}}$. Proposed controllers for position control of the group are (5) and (6), and inner controllers are assumed to be (30) and (33). In addition to stability proof that was represented in this chapter, a numerical problem simulation is used to show the effectiveness of proposed controllers.

## IV. Simulation

To show the great performance of the proposed controller, a formation control problem for formation flight of a group of 4 quadrotors is simulated. All dynamic equations of the quadrotor are simulated without any simplification. The group's agents have been ordered to fly along the way in a specified formation like Fig. 2. In addition, quadrotors are required to traverse the pass presented in (37). That is, virtual leader of the group has a movement like (37). Each quadrotor in the group should become aware of its neighbors and itself states. Communication graph in this paper is assumed to be as shown in fig. 3.

$$x_0 = 0.1\,t + \sin(t)$$
$$y_0 = 2 + \cos(t) \quad (37)$$

For quadrotor's position and angles measurement a number of different sensors are used. For altitude measurement, the sensor used is a barometer with 2% error. IMU with 2% error is for measurement of the Euler angles. For quadrotor's position measurement a combination of IMU and GPS with 1% error is used. These errors are simulated as measurement noise and white noise. For example, equation (38) is used as output of the $i_{th}$ quadrotor's barometer.

$$z_{i_measure} = z_i + 0.02 \times \text{rand}(0,1) \quad (38)$$

All measured states are affected by errors. In addition to these measurement noises, disturbances are considered in two directions of $x$ and $y$. Therefore, dynamic equations of the $i_{th}$ quadrotor in these two directions, are like (39) instead of (2).

$$\ddot{x}_i = f_{xi}(\phi_i, \theta_i, \psi_i, U_{1i}) + w_{xi} \quad (39)$$

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$$\ddot{y}_i = f_{yi}(\phi_i, \theta_i, \psi_i, U_{1i}) + w_{yi}$$

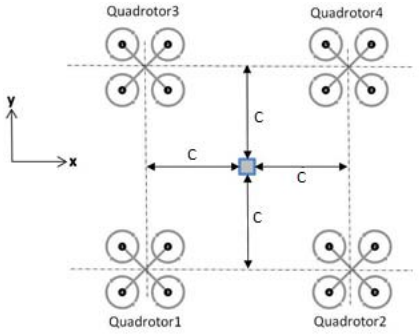



Fig. 2. Desired formation for quadrotors moving

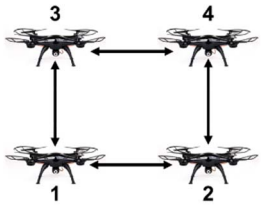



Fig. 3. Communication graph used in group

TABLE I. CONSTANT MATRICES USED FOR KALMAN FILTER

| Value | Parameter |
|---|---|
| *diag* (5) | $\boldsymbol{Q_{12\times12}}$ |
| *diag* (100) | $\boldsymbol{R_{12\times12}}$ |
| *diag* (0.5) | $\boldsymbol{A_{12\times12}}$ |
| *diag* (1) | $\boldsymbol{H_{12\times12}}$ |
| *diag* (1) | $\boldsymbol{I_{12\times12}}$ |

Where $w_{xi}$ and $w_{yi}$ are applied disturbances to the $i_{th}$ quadrotor in $x$ and $y$ directions respectively. These disturbances are simulated as random noise. Therefore, this multi-agent system has measurement noise and disturbances together. In order to overcome this problem and because presented control protocols require exact states values, Kalman filter is used. Constant matrices used in Kalman filter are shown in Table 1. Each agent in system have 12 states, so constant matrices are $12 \times 12$. In order to shed lights on this issue, two different situations are simulated. First simulation is the system with measurement noises without disturbance. Second simulation is the system with measurement noise and disturbances together. Movement of $1_{st}$ quadrotor in $x$ and $y$ directions are separately shown in fig. 4 and fig. 5 when there is no disturbance, and are been shown in fig. 6 and fig. 7 when there are both measurement noise and disturbances in the system.

Comparing fig. 4 with fig. 6, and fig. 5 with fig. 7 it is obvious that when just measurement noise affects the system, Kalman filter estimates states accurately. But this estimation is not that accurate when there are both measurement noise and disturbances in the system. Fig. 8 is a magnification of fig. 6, to show the measurement noise better.

Performances of the proposed control protocols are shown in fig. 9 to fig. 17. It is obvious that these protocols are chosen properly. Fig. 15 shows the defined error value of the $1_{st}$ quadrotor and the $4_{th}$ quadrotor. Error of the quadrotors are calculated through (9). These errors could not be always zero because of the formation flight and a distance that exist between the quadrotors. Moreover, fig. 15 shows the excellent performance of the proposed protocols. Fig. 17 shows formation of the group in different times. This figure indicates that the quadrotors always have the formation like the desired formation shown in fig. 2.

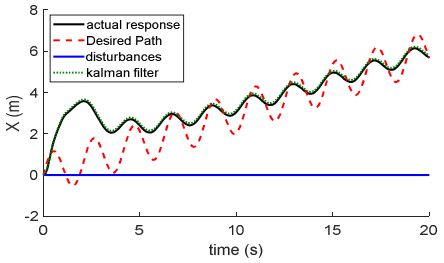



Fig. 4. Tracked path by 1st quadrotor in the absence of disturbance in X direction

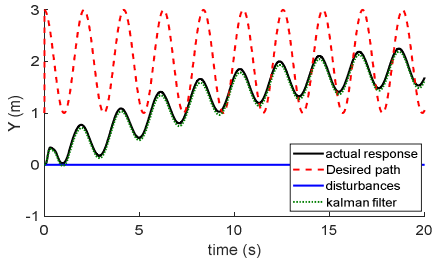



Fig. 5. Tracked path by 1st quadrotor in the absence of disturbance in Y direction

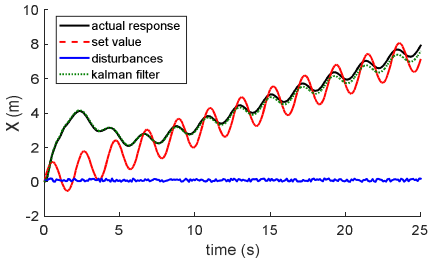



Fig. 6. Tracked path by 1st quadrotor in the presence of disturbance in X direction

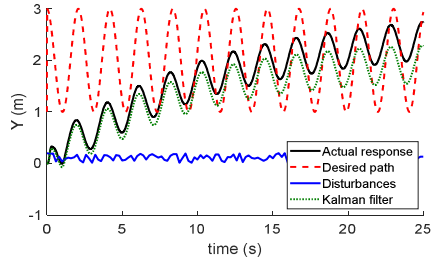



Fig. 7. Tracked path by $1_{st}$ quadrotor in the presence of disturbance in Y direction

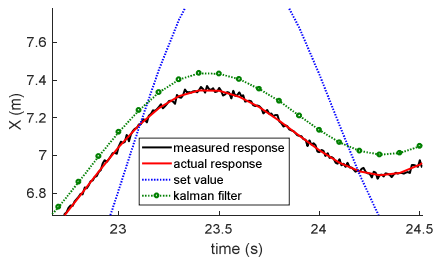



Fig. 8. Zoom in figure 5 to show measurement noise and estimation error

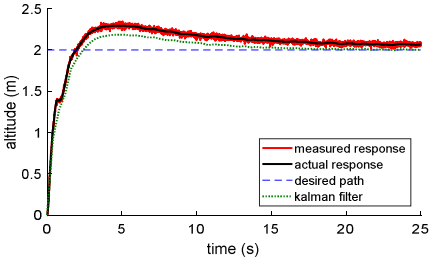



Fig. 9. Tracked path by 1st quadrotor in the presence of disturbance in Z direction

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TABLE II. CONSTANTS USED IN THE SIMULATION

| constant | value | constant | value |
|---|---|---|---|
| $r_1$ | 1 | $d$ | $1.1 \times 10^{-6}$ |
| $r_2$ | 2 | $b$ | $54.2 \times 10^{-6}$ |
| $k_{py1}$ , $k_{px1}$ | 0.587 | $L$ | 0.24 |
| $k_{py2}$ , $k_{px2}$ | 0.587 | $m$ | 1 |
| $k_{dy}$ , $k_{dx}$ | 0.483 | $g$ | 9.81 |
| $k_{Iy}$ , $k_{Ix}$ | 0.017 | $I_x$ | $8.1 \times 10^{-3}$ |
| $N$ | 4 | $I_y$ | $8.1 \times 10^{-3}$ |
| $n$ | 2 | $I_z$ | $14.2 \times 10^{-3}$ |
| $g$ | 9.81 | $I_R$ | $10.4 \times 10^{-5}$ |
| $\Delta_y$ | [0 0 1 1] | $\Delta_x$ | [0 1 0 1] |

Performances of the proposed control protocols are shown in fig. 9 to fig. 17. It is obvious that these protocols are chosen properly. Fig. 15 shows the defined error value of the $1_{st}$ quadrotor and the $4_{th}$ quadrotor. Error of the quadrotors are calculated through (10). These errors could not be always zero because of the formation flight and a distance that exist between the quadrotors. Moreover, fig. 15 shows the excellent performance of the proposed protocols. Fig. 17 shows formation of the group in different times. This figure indicates that the quadrotors always have the formation like the desired formation shown in fig. 2.

## V. CONCLUSION

The presence of a leader for a group, despite its advantages, can have many disadvantages. Failure to perform a mission when a problem occurs for the group leader is one of these disadvantages. Therefore, in this paper, virtual structure was used instead of leader-follower structure in formation flight problem. Dynamic equations of quadrotors were presented and used without any simplification. The group used two loops for flight controlling. Inner loop which was used for attitude control of agents, combines feedback linearization controller with PD controller. Outer loop which was designed for position control, uses a combination of consensus protocol with PID controller. Mission of the group, formation flight and path tracking simultaneously, was performed using these two control loops in the presence of both measurement noise and disturbances. Simulation results showed the excellent performance of the presented protocols for formation flight control of the group.

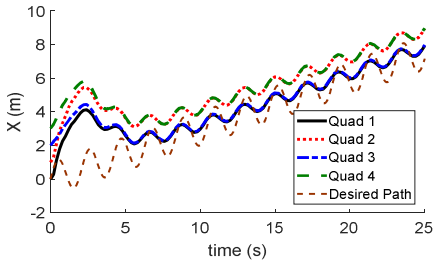



Fig. 10. Desired path and tracked path by the group in X direction

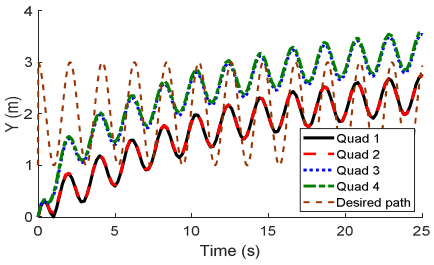



Fig. 11. Desired path and tracked path by the group in Y direction

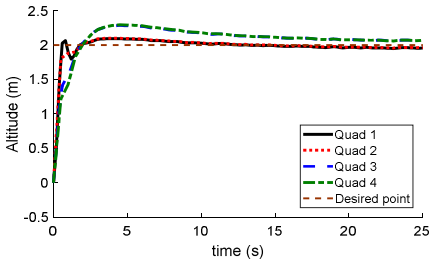



Fig. 12. Desired path and tracked path by the group in Z direction

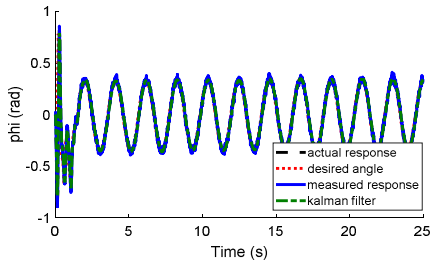



Fig. 13. Desired roll angle and tracked roll angle by 1st quadrotor

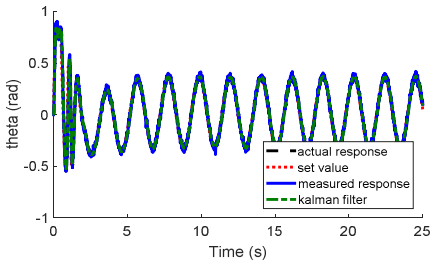



Fig. 14. Desired pitch angle and tracked pitch angle by 1st quadrotor

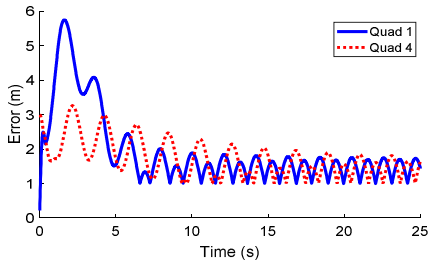



Fig. 15. 1st and 4th quadrotors error in the mission

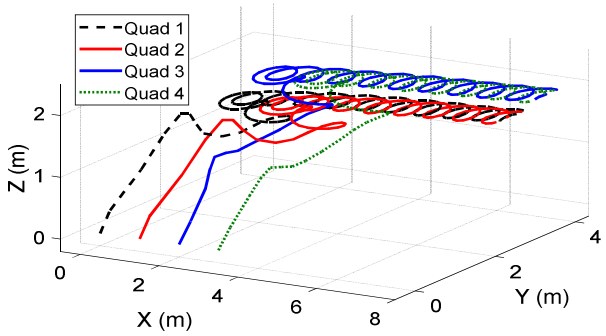



Fig. 16. Tracked path by the group in 3D

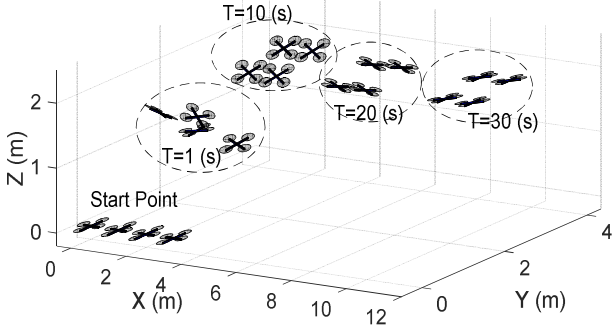



Fig. 17. Quadrotors formation along passing the mission

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